

Appendix C Overview

Appendix C.1 Linear Systems in 2 variables (2 days)

2 equations, 2 unknown variables
to be solved simultaneously
Also called 2×2 system.

Appendix C.2 Systems of Linear Equations in 3 variables (3 days)

3 equations, 3 unknown variables
to be solved simultaneously
Also called 3×3 system.

Math 60 App C.1 2x2 Systems

- Objectives:
- 1) Determine if an ordered pair is a solution of a 2x2 system
 - 2) Solve a 2x2 system by graphing
 - 3) Solve a 2x2 system by substitution
 - 4) Solve a 2x2 system by elimination

All of these objectives are a review of Math 45.

- ① Determine if the ordered pair is a solution of the system

$$\begin{cases} 2x + y = 2 & \textcircled{A} \\ 4x - 3y = 9 & \textcircled{B} \end{cases}$$

a) $(-2, 6)$

To be a solution of the system, the ordered pair must make both equations true.

Step 1: substitute $x = -2$ and $y = 6$ into top equation. ①

$$2(-2) + 6 = 2$$

$$-4 + 6 = 2$$

$$2 = 2.$$

Result is true, continue to next step.
(If result were false, we could stop and write "no.")

Step 2: substitute $x = -2$ and $y = 6$ into bottom equation ②

$$4(-2) - 3(6) = 9$$

$$-8 - 18 = 9$$

$$-26 \neq 9$$

Result is false, so $(-2, 6)$ is NOT a solution of both equations, and not a solution of system.

Step 3: Write "yes" or "no"

NO

Math 60 App C.1 2x2 systems

b) $(\frac{3}{2}, -1)$

substitute into top equation (A)

$$2\left(\frac{3}{2}\right) + (-1) = 2$$

$3 - 1 = 2$, continue to next equation.

substitute into bottom equation (B)

$$4\left(\frac{3}{2}\right) - 3(-1) = 9$$

$$2 \cdot 3 + 3 = 9$$

$$6 + 3 = 9.$$

YES both equations are true for $(\frac{3}{2}, -1)$.

(2) Solve the system by graphing.

* CAUTION * You should use graph paper

$$\begin{cases} x + y = 1 & \textcircled{C} \\ -2x + y = 7 & \textcircled{D} \end{cases}$$

step 1: Graph top equation (C) neatly, on graph paper.

$$x + y = 1$$

$$y = -x + 1$$

slope $m = -1$

y-int $b = 1 \Rightarrow (0, 1)$

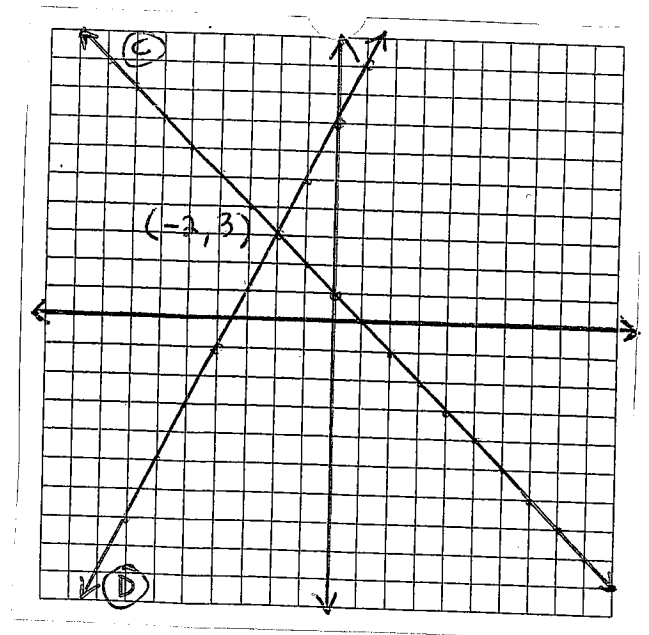
step 2: Graph bottom equation D

$$-2x + y = 7$$

$$y = 2x + 7$$

slope $m = 2$

y-int $b = 7 \Rightarrow (0, 7)$



step 3 Use grid to identify point of intersection = solution.

$(-2, 3)$

step 4: Optional: Check by substituting in both equations.

③ Solve the system by substitution

$$\begin{cases} 3x + y = -9 & \textcircled{E} \\ -2x + 3y = 17 & \textcircled{F} \end{cases}$$

step 1: Select an equation and solve for a variable.
Choose wisely!

choose $\textcircled{E}$, solve for x requires $\div 3$ and fractions
BEST CHOICE * choose $\textcircled{E}$, solve for y no fractions

choose $\textcircled{F}$, solve for x requires $\div (-2)$ + fractions

choose $\textcircled{F}$, solve for y requires $\div 3$ and fractions

$$\begin{array}{r} 3x + y = -9 \\ -3x \quad -3x \\ \hline \end{array}$$

$$y = -3x - 9$$

step 2: Substitute result from step 1 into the other equation.

If you chose $\textcircled{E}$, substitute into $\textcircled{F}$ } This is how we
If you chose $\textcircled{F}$, substitute into $\textcircled{E}$ } use both equations
to solve.

$$-2x + 3(\underbrace{-3x - 9}_{\text{replace } y}) = 17$$

step 3: Solve for first value.

$$-2x - 9x - 27 = 17$$

dist

$$-11x - 27 = 17$$

combine

$$+27 \quad +27$$

$$\frac{-11x}{-11} = \frac{44}{-11}$$

isolate

$$x = -4$$

step 4: substitute result into either eqn, or result from step 1.

$$y = -3x - 9$$

$$y = -3(-4) - 9$$

$$y = 12 - 9$$

$$y = 3$$

step 5: Write ordered pair

$$\boxed{(-4, 3)}$$

always x first!

④ Solve the system by elimination.

$$\begin{cases} 4x - 5y = 16 & \textcircled{G} \\ 2x + 3y = -3 & \textcircled{H} \end{cases}$$

step 1: select a variable to eliminate

BEST CHOICE * option 1: eliminate x

find LCM of coefficients x : 4x and 2x LCM = 4
mult $\textcircled{H}$ by -2 to get -4x.

option 2: eliminate y

find LCM of coefficients y : -5y and 3y LCM = 15
mult $\textcircled{G}$ by 3 to get -15y
mult $\textcircled{H}$ by 5 to get 15y

step 2: Multiply one or both equations by numbers needed to get (LCM) and (- LCM) for coefficients of variable to be eliminated.

$\textcircled{H}$ by -2

$$\begin{aligned} -2(2x) + (-2)(3y) &= (-2)(-3) \\ -4x - 6y &= 6 & \textcircled{H} \text{ new} \end{aligned}$$

mult all terms
both sides

step 3: Add like terms of the equations to eliminate your chosen variable

$$\begin{array}{rcl} 4x - 5y & = & 16 & \textcircled{G} \\ -4x - 6y & = & 6 & \textcircled{H} \text{ new} \\ \hline 0 & -11y & = & 22 \end{array}$$

step 4: Solve for remaining variable

$$\begin{aligned} \frac{-11y}{-11} &= \frac{22}{-11} \\ y &= -2 \end{aligned}$$

step 5: Substitute back into either equation to find other value.

$$\begin{aligned} \Rightarrow \textcircled{H} \quad 2x + 3(-2) &= -3 \\ 2x - 6 &= -3 \\ 2x &= 3 \\ x &= 3/2 \end{aligned}$$

step 6: Write ordered pair

$$\boxed{(3/2, -2)}$$

Extra Practice: Which method is easiest? Use it to solve.

$$\begin{cases} x + 3y = 0 & \textcircled{A} \\ -2x + 4y = 30 & \textcircled{B} \end{cases}$$

substitution

$$x = -3y$$

$$-2(-3y) + 4y = 30$$

$$6y + 4y = 30$$

$$10y = 30$$

$$y = 3$$

$$x = -3(3)$$

$$x = -9$$

$$\boxed{(-9, 3)}$$

elimination

$$2x + 6y = 0$$

$$-2x + 4y = 30$$

$$10y = 30$$

$$y = 3$$

$$x + 3(3) = 0$$

$$x + 9 = 0$$

$$x = -9$$

$$\boxed{(-9, 3)}$$

$$\begin{cases} 2x - 4y = 18 & \textcircled{C} \\ 3x + 5y = -3 & \textcircled{D} \end{cases}$$

substitution: divide $\textcircled{C}$ by 2

$$x - 2y = 9$$

$$x = 2y + 9$$

$$3(2y + 9) + 5y = -3$$

$$6y + 27 + 5y = -3$$

$$11y + 27 = -3$$

$$11y = -30$$

$$y = -\frac{30}{11}$$

$$x = 2\left(-\frac{30}{11}\right) + 9$$

$$x = -\frac{60}{11} + \frac{99}{11}$$

$$x = \frac{39}{11}$$

$$\boxed{\left(\frac{39}{11}, -\frac{30}{11}\right)}$$

elimination: elim x

$$3[2x - 4y] = 18 \cdot 3$$

$$-2[3x + 5y] = -3 \cdot 2$$

$$6x - 12y = 54$$

$$-6x - 10y = 6$$

$$-22y = 60$$

$$y = \frac{60}{-22} = -\frac{30}{11}$$

$$2x - 4\left(-\frac{30}{11}\right) = 18$$

$$2x + \frac{120}{11} = 18$$

$$2x = 18 - \frac{120}{11} = \frac{78}{11}$$

$$x = \frac{1}{2} \cdot \frac{78}{11} = \frac{39}{11}$$

$$\boxed{\left(\frac{39}{11}, -\frac{30}{11}\right)}$$